Name: ______________________________________ Date: ___________________

A2CC Summation Notation

Much of our work in this unit will concern **adding the terms of a sequence**. In order to specify this addition or summarize it, we introduce a new notation, known as **summation or sigma notation** that will represent these sums. This notation will also be used later in the course when we want to write formulas used in statistics.

SUMMATION (SIGMA) NOTATION

$$\sum_{i=a}^{n} f(i) = f(a) + f(a+1) + f(a+2) + \cdots + f(n)$$

where ***i*** is called the **index variable**, which starts at a value of a, ends at a value of n, and moves by unit increments (increase by 1 each time).

***Exercise* #1:** Evaluate each of the following sums.

(a) $\sum_{i=3}^{5} 2i$

(b) $\sum_{k=-1}^{3} k^2$

(c) $\sum_{j=-2}^{2} 2^j$

(d) $\sum_{i=1}^{5} (-1)^i$

(e) $\sum_{k=0}^{2} (2k+1)$

(f) $\sum_{i=1}^{3} i(i+1)$

***Exercise* #2:** Which of represents the value of $\sum_{i=1}^{4} \frac{1}{i}$?

(1) $1/10$

(3) $\frac{25}{12}$

(2) $9/4$

(4) $\frac{31}{24}$

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***Exercise* #3:** Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of $\sum_{i=4}^{7} a_i$

It is also good to be able to place sums into sigma notation. These answers, though, will not be unique.

***Exercise* #4:** Express each sum using sigma notation. Use *i* as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a) $9+16+25+\cdots+100$ (b) $-6+-3+0+3+\cdots15$ (c) $1/25 + 1/5 + 1 + 5 + \cdots 625$

***Exercise* #5:** Which of the following represents the sum $3+6+12+24+48$?

(1) $\sum_{i=1}^{5} 3^i$ (3) $\sum_{i=0}^{4} 6^{i-1}$

(2) $\sum_{i=0}^{4} 3(2)^i$ (4) $\sum_{i=3}^{48} i$

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***Exercise* #6:** Some sums are more interesting than others. Determine the value of $\sum_{i=1}^{99}\left(\frac{1}{i}-\frac{1}{i+1}\right)$. Show your reasoning. This is known as a **telescoping series (or sum)**.

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A2CC Summation Notation

HOMEWORK

1. Evaluate each of the following. Place any non-integer answer in simplest rational form.

(a) $\sum_{i=2}^{5} 4i$

(b) $\sum_{k=0}^{3} \left(k^2+1\right)$

(c) $\sum_{j=-2}^{0} \left(2j+1\right)$

(d) $\sum_{i=-1}^{3} 2^i$

(e) $\sum_{k=0}^{2} \left(-1\right)^{2k+1}$

(f) $\sum_{i=1}^{3} \log\left(10^i\right)$

(g) $\sum_{n=1}^{4} \frac{n}{n+1}$

(h) $\dfrac{\sum_{i=2}^{4} \left(i+1\right)^2}{\sum_{i=2}^{4} \left(i^2+1\right)}$

(i) $\sum_{k=0}^{3} 256^{\frac{1}{2^k}}$

2. Which of the following is the value of $\sum_{k=0}^{4} \left(4k+1\right)$?

(1) 53 (3) 37

(2) 45 (4) 80

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3. The sum $\sum_{i=4}^{7} 2^{i-7}$ is equal to

(1) $15/8$ (3) $3/4$

(2) $3/2$ (4) $7/8$

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4. Write each of the following sums using sigma notation. Use k as your index variable. Note, there are many correct ways to write each sum (and even more incorrect ways).

(a) $-2+4+-8+\cdots+64+-128$ (b) $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{81}+\frac{1}{100}$ (c) $4+9+14+\cdots+44+49$

5. Which of the following represents the sum $2+5+10+\cdots+82+101$?

(1) $\sum_{j=1}^{6}(4j-3)$ (3) $\sum_{j=1}^{10}(j^2+1)$

(2) $\sum_{j=3}^{103}(j-2)$ (4) $\sum_{j=0}^{11}(4^j+1)$

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6. A sequence is defined recursively by the formula $b_n=4b_{n-1}-2b_{n-2}$ with $b_1=1$ and $b_2=3$. What is the value of $\sum_{i=3}^{5}b_i$? Show the work that leads to your answer.

REASONING

6. A curious pattern occurs when we look at the behavior of the sum $\sum_{k=1}^{n}(2k-1)$.

(a) Find the value of this sum for a variety of values of n below:

$n=2$: $\sum_{k=1}^{2}(2k-1)=$ $n=4$: $\sum_{k=1}^{4}(2k-1)=$

$n=3$: $\sum_{k=1}^{3}(2k-1)=$ $n=5$: $\sum_{k=1}^{5}(2k-1)=$

(b) What types of numbers are you summing? What types of numbers are the sums?

(c) Find the value of n such that $\sum_{k=1}^{n}(2k-1)=196$.